On Graduating the Thickness of Violin Plates to Achieve Tonal Repeatability

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Abstract

The various criteria used for graduating the thickness of violin plates are discussed. These are shown apparently to be conflicting. A case is made that the flexural stiffness of the plates should be given priority. A simple indirect method is given for finding the flexural stiffness of the plates without the need to flex them in the hands or to apply forces and measure deflections. The method requires the back and belly resonance frequencies in the "ring" and "X" modes (modes 2 and 5) to be combined and modified by the plate weight to find a number that has proportionality to the plate stiffness. A suggested value is given for this stiffness number. The same stiffness number can be used for violins, violas and cellos.

The forces applied to the body by the string

Before discussing plate thickness graduation, it is necessary to have a clear understanding of how the transversely vibrating string drives the violin. The bowed violin string assumes a waveform that approximates to a saw tooth shape. This can be represented as a series of simple harmonic vibrations of displacement amplitude a_n that decline as 1/n. The transverse string vibration (referred to as TSV) puts a transverse force (TSV force) on the bridge, which tends to rock it in its own plane. The transverse displacement of the string increases its tension and causes a vibration of string tension (referred to as longitudinal string vibration, or LSV). The LSV force puts a downward force on the bridge, which tends to make the bridge bounce vertically in its own plane and the violin bend in its length. Experimental work shows that the bowed string induces LSV at all harmonics [1]. The ratio LSV force/TSV force, as applied to the bridge, varies as the string displacement, but for any specific string displacement it is constant at all harmonics. For a string bowed moderately loudly (assuming a saw tooth wave) this ratio is ~0.04. Although the LSV force is small the body has a higher admittance to it than to the TSV force. The transfer of power from the string to the body, per unit force, is about ten times higher. So the bowed string loses a significant amount of its energy to the violin through

LSV. LSV that results from the transverse vibration of the string, I call primary LSV. It has a non-linear relationship with TSV and only occurs in the bowed string.

This distinguishes it from secondary LSV that is generated by dynamic activity in the body and occurs in all four strings. The motions of the violin body can alter the distance between the string supports (the nut, the bridge and the saddle) and generate significant amounts of secondary LSV, which can flow from the string to the body and from the body to the string, at the bridge, the nut and the saddle. Secondary LSV has a linear relationship with the dynamic activity in the body. But, since the dynamic activity of the body results in part from the effects of primary LSV, secondary LSV does not have a linear relationship with the TSV. The vibrations of the string and body are much more closely coupled through LSV than TSV.

The response of the violin to LSV forces has been shown to be very sensitive to the arching shape. The generation of LSV and the role of the plate arching in the coupling of LSV to the body is more fully explained and justified in a paper I have written [2].

The violin complete with its strings vibrates in patterns called modes. The modal displacements peak at resonance frequencies. At any one frequency the vibrating shape will in general be made up of a combination of several overlapping modes. An alternating force applied to a body, will best excite those modes that are close to the force in frequency and have large modal displacements in the direction of the force. Therefore, the spatial arrangement of the applied forces is important. A force that is applied transversely to the bridge may not excite the same combination of modes as a force applied vertically at the bridge.

Violin behavior is often characterized by the *frequency response function (frf)* of the radiated sound. To find the frequency response function, the bridge is struck transversely by a pendulum and the radiated sound is analyzed and divided by the analyzed force applied to the bridge. The resulting spectrum shows at all frequencies how strongly the violin radiates sound from unit force applied transversely at the bridge. It is assumed that the system is linear. This has been investigated and shown to be so [4]. This means that the mode shapes and frequencies found will apply at any level of excitation. When the string is bowed, the system as a whole will be non-linear. The operating shape (combination of modes) will be specific to the level of excitation. The *frf*, at best, describes violin response to a TSV force. It cannot be assumed that it describes the body's response to primary LSV forces. The unanswered question is, if ever it were possible to have two violins of matching *frf* and mode shapes, would they sound the same when excited by a bow? Perhaps they would, but I don't think the science has been done. It may be overoptimistic to think of the *frf* as an "acoustic signature."

The difficulties of plate thickness graduation

In making the wooden parts of a violin there are really only two things to get right, the shape and the thickness. We tend to think of the arching shape as being the shape of the outside surface. Acoustically, the shape of the arching of the plates is the shape of the centerline of the wood thickness. If we make the arching with the same outer surface shape in all our instruments but graduate the thicknesses differently, then the wood-centerline arching shape will be different each time. This difference is not as inconsequential as is often assumed, and is certainly sufficient to affect significantly how the body responds to LSV forces.

To make progress in violinmaking it is very important to alter one thing only from one violin to another in order to test its effect on tone, or to change nothing if we want to repeat the same tone every time. A series of violins with differing wood but the same shapes and thicknesses, will not produce the same result. Ideally, we must find a way of adjusting the thicknesses of the wood in such a way that the wood-centerline arching shape, the plate flexural stiffness and the resonance frequencies are maintained constant.

Suppose one makes a violin and then removes the belly and adjusts the thicknesses and reassembles the instrument until the tone is optimized. In doing this the maker will have altered the wood-centerline arching shape, the resonance frequency, the flexural stiffness of the plate to different degrees along and across the grain, and the mass. To attribute the resulting tonal change, to a change in resonance frequency alone (or any other single variable) is not justified, and the maker will have learned nothing from the exercise.

A violin made as a copy of another cannot be the same unless the wood thicknesses are adjusted to give the same resonances and the same plate flexural stiffness, and the surface shape of the arching is adjusted to give the same wood-centerline shape with the different wood thicknesses.

Criteria for graduating violin plate thickness

There are at least four basic systems of plate thickness graduation that have been used, and are still used. They rely on different criteria, a case can be made in support of all of them, and they all have their advocates.

Criterion 1. *We should make the wood thickness the same every time.*

It is very important tonally that the same wood-centerline arching shape should be reproduced from violin to violin. This can most conveniently be achieved by making the plate thicknesses the same for every instrument. This method ignores the requirements of the valid criteria 2 and 3 below, thus detracting from its undoubted benefits.

In a paper that I will submit to this Journal shortly [3], I show that by adjusting the surface arching shape, it is possible to make the plate thicknesses different from instrument to instrument and still maintain the same wood-centerline arching shape.

Criterion 2. *We should adjust the thickness of the wood to make the long-grain and cross-grain plate bending stiffness constant.*

The radiation of sound requires that the wood flex. The stiffness of the plate in flexure, both along and across the grain will need to be the same for all instruments made in order that the plates behave in the same way. There is no point in having identical arching shapes if the flexural stiffnesses are different. To ensure repeatable tonal results the wood thickness must be adjusted to give the same long-grain and cross-grain flexural stiffness. An instrument has a certain playing resistance that the player is sensitive to. Some violins speak too easily and others need really hard playing to respond. There are a large number of possible causes of this but the flexural stiffness of the plates must be involved here. Historically, many makers may have removed wood from the detached plates until some degree of flexibility was achieved. They probably assessed the flexural stiffness of a detached plate by pressing the thumbs into the center of the plate while at the same time pulling up on the edges with the fingers. Assessment of plate stiffness by feel may be reasonably accurate if a reference plate is available for comparison, but a less subjective method is presented in this paper.

Criterion 3. *We should adjust the natural resonance of the free plates to certain predetermined frequencies.*

The violin radiates sound from modes and it has been suggested that the frequency at which some of the major modes occur may be tonally significant. With this aim in view, Carleen Hutchins proposed her well-known method of plate tuning [5-7]. The thicknesses of the detached back and belly plates are graduated to bring the resonance frequencies of up to three of the free plate modes, the 1st, 2nd (or "X mode") and 5th (or "ring mode") to certain prescribed frequencies and certain relationships between the back and belly. The objective is that when these plates are assembled into an instrument it will produce a more even "loudness" across the instrument. My experience with tuning the detached plates to prescribed frequencies is that there is a tonal benefit. The balance between the upper and lower strings is affected. To that extent I find the method works. However, the problem with it is that plates of widely varying mass can all be tuned to the same resonance frequencies. These plates would all have different flexural stiffness and the requirement of criterion 2 would not be met. Joseph Curtin [8] pointed out his concern that thicknessing by resonance frequency alone, disregards other plate properties that seemed too important to neglect. My experience is that because of its failure to control the flexural stiffness of the plates, plate tuning to predetermined frequencies alone, does not give tonal consistency from one violin to another.

Criterion 4. *We should adjust the plate thicknesses of the assembled violin to match some of the principal modal frequencies and shapes to those of a recognized standard as revealed by the frequency response function (frf) of the radiated sound.*

This method was pioneered by Martin Schleske [9, 10] and is arguably a shift from plate tuning to body tuning. The surface arching shape can be taken from the instrument being copied but the subsequent adjustment of the thicknesses will alter the arching shape. The effect of the changes in arching on the way the body responds to LSV may not be picked up because of the absence of primary LSV in the excitation. There will be secondary LSV generated but it will stem from dynamic activity that does not result from the correct excitation. Advocates of this method might argue that the precise arching shape is unimportant because the mode shapes and frequencies are the ultimate concern. Many violinmakers may find this method frighteningly complicated.

Personally, I find the method too holistic, in that too many variables are altered at once. I like to be able to control separately the wood-centerline arching shape, the flexural stiffness and the resonance frequencies, so that I can identify their effects on the tone.

The criterion used in the method proposed in this paper

While it would be good to satisfy all the above criteria, that is clearly not possible. Choices have to be made. Essentially what I do is, graduate the plate thickness to maintain the same long-grain and cross-grain stiffness in all my instruments (criterion 2). I make this my priority because the coupling of the LSV with the body is highly dependent on the arching shape and the long-grain and cross-grain wood stiffnesses. I satisfy the plate-tuning requirement (criterion 3) by careful choice of wood. I satisfy the requirement of criterion 1 by modifying the surface arching shape to maintain the same wood-centerline arching shape with the thicknesses used.

I will now describe how I graduate the plate thickness to achieve prescribed plate flexural stiffness.

Determination of the plate flexural stiffness

The flexural stiffness of a detached violin plate is proportional to the product of its weight and the square of its resonance frequency. For those interested in the derivation of this basic relationship, it is given in an appendix to this paper. The relationship holds good for any size of plate, violin, viola or cello. The resonance frequency should be that of a mode that best samples the wood properties over a large area of the plate and in both directions of the grain. Of the various possible free-plate modes there are two that are particularly useful. These are traditionally described as the second and fifth modes, or as the ‘X mode’ and the ‘ring mode’. Many violinmakers are familiar with these modes and know how to find their resonance frequencies, or tap tones as they are often called. The methods need not be described here except to say that suspending the plate between the thumb and second finger, tapping it with a knuckle and comparing the modal frequency with a tuning fork or piano keyboard, achieves sufficient accuracy. The extra precision afforded by electronic methods is not needed but can be helpful to those with a less experienced ear. These procedures are well described by Hutchins [5]. If musical notation is used to define the tap tones, these must be converted to the frequency in

Hertz (cycles per second). We calculate the plate stiffness factor using the formula:

$$K = W\left(\frac{f_{\text{ring mode}} + f_{\text{X mode}}}{2}\right)^2. \tag{1}$$

The calculated result is not an absolute stiffness but a figure (in grams) that is proportional to the stiffness. The procedure of taking the frequency as the average of the ring mode and X-mode frequencies needs some justification. The ring-mode frequency is largely determined by the thickness in the upper and lower bouts and can be lowered by thinning in these areas. It reflects the stiffness of the plate in bending along the grain. The X-mode frequency is largely determined by the thickness in the center bouts and can be lowered by reducing this. It reflects the stiffness of the plate in cross-grain bending. Combining both these modes in the formula therefore takes account of the effect of the wood bending resistance both along and across the grain. I graduate the plate to place the ring-mode frequency at an octave above the X-mode frequency. The difference is not important, but it must be consistent because it determines the ratio of the long-grain and cross-grain bending stiffnesses. I have found after applying this method over about 200 instruments (violins, violas and cellos) that tonally they are about twice as sensitive to the ring mode as the X mode. Since the ring mode and X-mode frequencies are about an octave apart the average calculated in the formula above gives a weighted preference to the ring mode because it has double the frequency of the X mode.

The formula may be applied to violins, violas or cellos equally. In the case of the bigger instruments the resonance frequency will be much lower than that of the violin, but the weight of the plates will be much greater. The resulting stiffness factor may be the same in all cases. The fact that it gives consistent results regardless of whether the instrument is a violin or a cello does support the claim that it can be used reliably to compare one violin with another.

The procedure

To graduate the thickness of a back using this method, the center bouts and end bouts are reduced in thickness until the ring mode frequency is approximately an octave above the X-mode frequency. The actual graduation of the plate thicknesses within the center bouts and within the end bouts can be proportionate to a recognized standard. The graduations used by Stradivari, as shown by Sacconi [11], are a useful recognized standard for this purpose, but any reasonable system will do.

The main thing is that a similar system is used each time, in order to get consistency from one instrument to another. The plate is then weighed. The plate stiffness factor is the plate weight multiplied by the square of the average of the ring- and X-mode frequencies.

To graduate the thickness of a belly, the method is similar. The stiffness factor can be calculated at stages during the thickness graduation process (the stiffness is assessed before the sound holes or bass bar are done, the purfling is in, the fluting cut but the edges are square and the plate is maintained in an atmosphere of 55% relative humidity). In our workshop we graduate bellies with a uniform thickness all over, but any consistent system can be used. Through experience we have found that there is no need to achieve any specific relationship between the ring- and X-mode frequencies for the belly. This may be due to the cross grain bending in the center bouts being drastically reduced by the later cutting of the sound holes.

Now the question is, what is a good value for the plate stiffness factor? This will depend on what the maker wants to achieve tonally, the toughness or resistance of the violin under the bow and other factors. It will also depend to some extent on the arching shape. In our workshop we have found that a value of stiffness factor of 4,250,000 for bellies and 7,250,000 for backs (plate weight in grams, frequency in Hertz) works well, resulting in wood thicknesses that average a little more than those given by Sacconi for Stradivari [11]. Maintaining constant plate stiffness from violin to violin by the use of this formula will take account of varying wood properties, but the actual stiffness factor used should be found by each maker to suit his/her arching shape, varnish, tonal preference, and playing feel. The same value of stiffness factor can be used for violins, violas, and cellos.

Meeting resonance criteria also

Having graduated the thickness of the plates to give constant flexural stiffness (criterion 2), I will show how we aim for constancy in resonance (criterion 3). We have found that violins made from plates of constant wood-centerline arching shape and with thickness graduated to give constant plate stiffness will be very similar tonally and will have the same resistance or ease of response to the player. Because we have not tuned the plates to constant resonance frequencies, there could be differences in the brightness of the sound and evenness. When the violin is new, plates that achieved their stiffness from a high resonance frequency and a low weight do sound a little brighter than those with plates that achieved the stiffness with more weight and lower resonance frequency. They also are more ‘alive’ on the top string and more ‘reluctant’ on the lowest string. The reverse is true for plates of low resonance frequency and high weight. After the violin has been played for a while (~1 month) these differences become much less noticeable. One could attempt to rectify a high resonance-low weight characteristic by reducing the plate stiffness below the chosen standard, but to do so would have an immediate effect on the tone by altering the plate stiffness. When it comes to a choice between achieving a certain resonance frequency and achieving a certain plate stiffness number, I would certainly go for the plate stiffness.

However, we do recognize the effects of resonance criteria by choosing our wood carefully. Our experience has shown that the brightness of tone of the instrument and the balance between the top and bottom strings does depend much more on the wood used in the belly than that in the back. Provided that the belly stiffness is always close to 4,250,000 and to 7,250,000 for the back, the balance is characterized by a factor calculated as twice the belly weight plus the back weight. We have found from experience that if this ‘balance factor’ is ~240, the balance between the top and bottom strings will be good. A balance factor of ~240 very probably places the wood resonance (body mode) peak well, relative to the air resonance peak, thus satisfying a recommendation of Hutchins [5]. A high balance factor (~253) will favor the lower strings, and a low balance factor (~227) will favor the upper strings. Some variation away from 240 is not a problem. Some players actually prefer the violins with a high balance factor while others prefer those with a low balance factor.

A particular balance factor can be achieved by having either a light belly and heavy back, or a light back and a heavy belly, or a medium-weight back and belly. Again from experience, we have found that our tonal aspirations are best met by having a lightweight, high-frequency belly and a heavyweight, low-frequency back. The light belly and heavy back combination seems to both brighten and deepen the tonal quality of the instrument, giving the tone more breadth. For this reason, we choose our belly wood to aim for a high stiffness-to-weight ratio and backs with a low stiffness-to-weight ratio. For each instrument we record a ‘quality factor’ that is calculated as the weight of the back divided by the weight of the belly. We attempt to keep this quality factor high. Generally, the more deeply figured maple needs to be relatively thicker in the end bouts and a touch thinner in the center bouts, but overall this results in a heavier back than for a less-figured back. We have found that, given a choice of our instruments, players will choose those with a high ‘quality factor’.

Some typical figures

The table below shows the plate weights and resonance frequencies averaged over about 150 instruments made in our workshop to the thickness graduation method given in this paper. Where figures are shown in brackets, they indicate the highest and lowest figures included in the average, to give some idea of the range. The average figures are arrived at by calculation and, therefore, show an unjustified number of significant figures.

Table 1. Mean values of weights, resonance frequencies and derived stiffness, balance, and quality factors for back and belly plates attained by using flexural stiffness as the primary criterion for thickness graduation.

	Violin	**Viola**	**Cello**
Back weight	109 g [low 101, high 117]	148 g [low 135, high 164]	673 g [low 630, high 716]
Back ring-mode freq.	345 Hz	302 Hz	142 Hz
Back X-mode freq.	171 Hz	140 Hz	66 Hz
Back stiffness factor	7,255,000	7,228,000	7,279,000
Belly weight	65 g [low 59, high 71]	97 g [low 88, high 106]	434 g [low 412, high 516]
Belly ring-mode freq.	360 Hz	296 Hz	148 Hz
Belly X-mode freq.	151.5 Hz	118 Hz	49 Hz
Belly stiffness factor	4,247,000	4,197,000	4,211,000
Balance factor	239 ($[65 \times 2] + 109$)	342 ($[97 \times 2] + 148$)	1543 ($[434 \times 2] + 673$)
Quality factor	1.677 ($109 \div 65$)	1.526 ($148 \div 97$)	1.551 ($673 \div 434$)

The validity of the method

The validity of the method can be appraised by asking several questions:

1. How does the logic of the philosophy involved stand up against alternative systems?

2. How valid is the theoretical derivation of the basic equation (given in the Appendix)?

3. Does the choice of modes used enable the formula to control reliably the long-grain and cross-grain plate stiffness?
4. What experimental evidence is there that the method achieves the consistency that is claimed for it?

So far as the first three points are concerned, I have set out the arguments and I leave it to the reader to answer the questions. Until such time as there is a reliable objective method for the evaluation of the tone and response of violins, we must rely on playing. I am very comfortable with that. If someone comes up with an alternative method of assessment and does the huge amount of testing needed to calibrate it, I will be very pleased.

Here are the results of my testing by playing. I made a large number of instruments always with the same wood-centerline arching shape. About ninety of these were graduated by tuning the detached plates to constant resonance frequencies. About 160 were graduated to maintain constant long-grain and cross-grain flexural stiffness. I can report that the inconsistency of the violins tuned by resonance frequency was so disappointing that I developed the method presented here. The result was a pronounced increase in consistency. I have explained the reason for the remaining small inconsistencies. These inconsistencies can be reduced by careful wood selection.

Of course, there is a scientific protocol for making subjective judgments. To do so would require me to submit a large number of violins graduated by this method, plus another equally large number of violins graduated by resonance frequency alone, to a number of players and ask them which group shows the greater consistency. I did not do this and I don't think this has been done for any of the proposed alternative methods of thickness graduation.

Conclusions

I am advocating that the thickness of detached plates of a violin should be graduated to achieve constant long-grain bending stiffness and constant cross-grain bending stiffness from instrument to instrument. Provided the wood-centerline shape of the arching is also maintained constant, the resulting instruments will have a degree of repeatability in the sound that is unachievable by any other system of plate thickness graduation. What differences in repeatability do remain can be minimized by careful wood selection, and the range of these variations lies well within the range of variation in player preferences.

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APPENDIX. Derivation of the relationship between stiffness, resonance frequency and weight

Imagine a strip of wood (say 1-inch wide) cut from the plate, which extends diagonally across the plate from one side to the other. If the strip is bent by applying a force to the center of the strip and pulling up on the ends, the deflection of the strip is given by, $\delta \propto \frac{\mathrm{PL}^3}{\mathrm{EI}}$, where P is the force applied to the strip at the center, L is the length of the strip, E is Young's modulus of the wood, and I is the second moment of area of the cross section of the strip. The resistance of the strip to bending is the stiffness of the strip. The stiffness K is defined as the force P required to produce unit deflection δ.

$$\therefore K \propto \frac{EI}{L^3} \tag{2}$$

If the strip is simply supported at both ends and is allowed to sag under its own weight, W, then again the stiffness of the strip is the weight W required to produce unit deflection δ. This stiffness K also complies with the relationship given by Eq. (2). The resonance frequency, in the first mode, of a mass hanging on the end of a spring is given by $f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta_{st}}}$, where Δ_{st} is the static extension of the spring under the weight of the mass. Similarly, if a lightweight elastic strip carries a mass at its center, this formula applies where Δ_{st} is the static deflection of the center of the strip. If the mass is distributed along the strip, the effective mass is reduced so the relationship would require a different constant. However, by eliminating constants we can write that the resonance frequency of the strip in the first mode is given by $f \propto \sqrt{\frac{g}{\Delta_{st}}}$. But $\Delta_{st} = \frac{5}{384}\frac{WL^3}{EI}$, where W is the weight of the beam (standard formula for the deflection of a uniformly loaded simply supported beam). By substitution and eliminating constants we can write that $f^2 \propto g\frac{EI}{WL^3}$, and therefore, $f^2 \propto \frac{1}{W}\frac{EI}{L^3}$.

But, from Eq. (2) $K \propto \frac{EI}{L^3}$, so by substitution and rearrangement, we obtain

$$K \propto Wf^2. \tag{3}$$

This says that the stiffness of the strip is proportional to the weight of the strip multiplied by the square of its resonance frequency.

Since this formula applies to a strip of any length, it would therefore apply to any and all strips of violin plate that one might consider. Consequently, the formula is applicable to the plate as a whole. Furthermore, the relationship is applicable equally to violins, violas and cellos. The jump from saying that the formula for the stiffness of a strip applies to the whole plate does overlook the stiffening effect of the arching shape. This again reminds us that the arching shape will affect the plate stiffness, and using a constant value of plate stiffness [as approximated by Eq. (3)] to get consistency between violins can only be done if the arching shape is unvarying.

However, it is my experience that, provided that the arching shapes are not greatly different, Eq. (3) can be used to produce violin plates of similar flexural stiffness.